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# Modelling the Distribution of Water-Stable Aggregates

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## ABSTRACT

The wet-sieving method using nested sieves is a common way to measure aggregate stability, but this method can only be used to measure the stability of whole soils, not the stability of individual aggregate size fractions. Thus, the main objective of this study was to develop a mechanistic model to estimate the amount of breakdown and distribution of aggregates in the usual wet-sieving method (using nested sieves). The amount of aggregate breakdown and its distribution in the various sieves are described in a series of equations. By using several key assumptions, these equations could be solved. The model was tested on several soil types of various textures and land use. For each soil, each of the six aggregate size fractions (5-8, 3-5, 2-3, 1-2, 0.5-1 and 0.3-0.5 mm) was wet-sieved separately to determine the actual breakdown and distribution of aggregates in the various sieves. The model showed good accuracy for several different soils despite not requiring any information about the properties of the soil or wet-sieving method. The mean estimation error of the model was 1.33 g (6.65%), and it also showed no bias in its estimation.

**Keywords: Aggregate breakdown, wet sieving, aggregate stability, nested sieves**

## INTRODUCTION

Wet-sieving using nested sieves (Yoder 1936; Kemper and Chepil 1965) is a common method to measure aggregate stability. This technique breaks down and separates the aggregates into various size fractions by sieving them through a series of graduated sieves under water. Nevertheless, this method can only be used to determine the stability of whole soils and not individual aggregate size fractions because, in a given sieve (except the uppermost sieve), there is a mixing of aggregates that were originally placed in that sieve (before wet-sieving) with the aggregates that had ruptured and fallen from the above sieves (after wet-sieving).

To determine the stability of individual aggregate size fractions means each aggregate size fraction has to be wet-sieved separately. In practice, however, separate wet-sievings are too tedious and time-consuming. Consequently, the objective of this study was to overcome these problems by developing a mathematical model to estimate the amount of breakdown and distribution of aggregates in the usual wet-sieving method (using nested sieves). By knowing the amount of breakdown in each aggregate size fraction, it would be possible to also determine their respective aggregate stabilities.

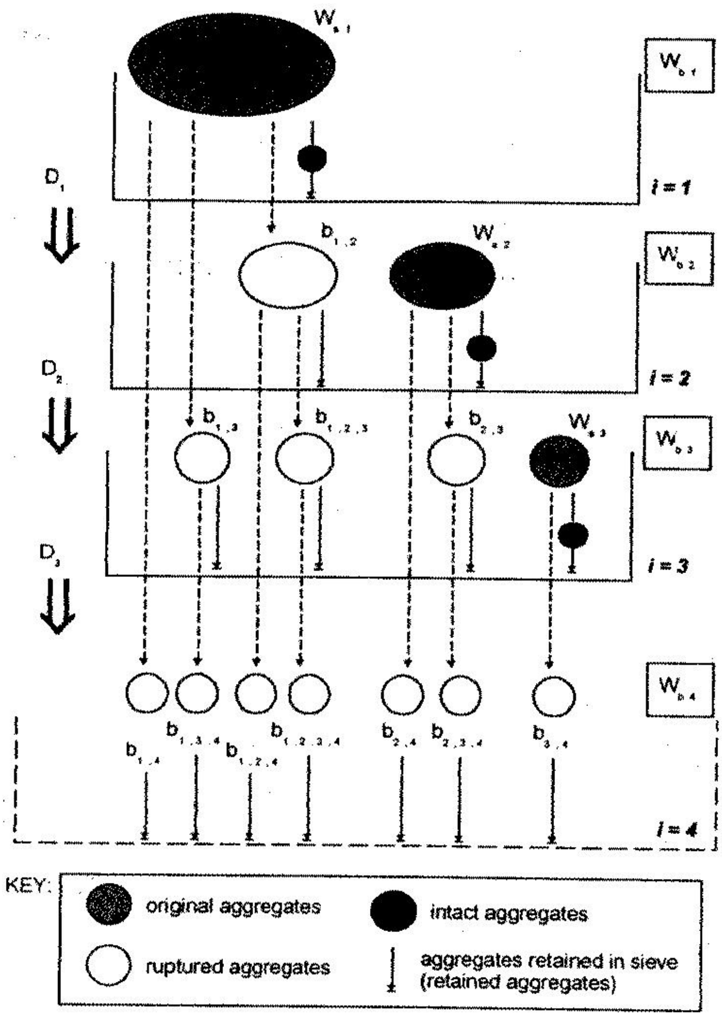


*Fig. 1: Simultaneous breakdown and movement of aggregates in a nest of three sieves in the wet-sieving method*

## THEORY

*Fig. 1* illustrates the breakdown and movement of aggregates in a nest of three sieves during wet-sieving, where $i$ denotes the sieve number ($i$ = 1 to $n$) where sieve 1 is the sieve with the largest aperture size, followed by sieve 2, and so on. Note that sieve $n$ (last sieve) refers to the container that holds the water and the nest of sieves.

### *Original Aggregates*

These are aggregates that were placed in each sieve before wet-sieving, and *ruptured aggregates* are those that had ruptured during wet-sieving (*Fig.1*). The weight of the original aggregates in sieve $i$ is $W_{ai}$, and the weight of all retained aggregates in sieve $i$ after wet-sieving is $W_{bi}$. The weight of ruptured aggregates from aggregate size fraction $i$ is $b_{i,i+1,i+2,...,n}$, where the subscripts denote the path of ruptured aggregates in the nested sieves. $b_{1,2}$, for example, is the weight of ruptured aggregates from sieve 1 in sieve 2; and $b_{1,2,3}$ is the weight of ruptured aggregates from sieve 1, settling in sieve 2 before further breaking down and settling in sieve

3. Note that $b_{i,i}$ (equal subscripts) denotes the weight of original aggregates in sieve $i$; for example, $b_{1,1}$ is $W_{a1}$, $b_{2,2}$ is $W_{a2}$, and $b_{3,3}$ is $W_{a3}$.

Aggregates retained in a given sieve after wet-sieving are referred as *retained aggregates*, and its weight $w_{i,\ldots,j}$ in sieve $j$ can be calculated by

$$w_{i,\ldots,j} = \begin{cases} b_{i,\ldots,j} - b_{i,\ldots,j,j+1} & (\text{for } i \neq j) \\ W_{ai} - \sum_{k=i+1}^{n} b_{i,k} & (\text{for } i = j) \end{cases} \tag{1}$$

where $w_{i,\ldots,j}$ is the portion of $b_{i,\ldots,j}$ or $W_{ai}$ remaining in sieve $j$ ($i \leq j$). For example, $w_{1,2,3}$ or $(b_{1,2,3} - b_{1,2,3,4})$ is the portion of $b_{1,2,3}$ that remained in sieve 3, and $w_{2,2}$ or $(W_{a2} - b_{2,3} - b_{2,4})$ is the portion of $W_{a2}$ that remained in sieve 2. Note that $w_{i,i}$ (equal subscripts) is the weight of the *intact aggregates* (*Fig.1*) and the weight of these aggregates must be known to determinate the stability of an aggregate size fraction. This is because the stability of aggregate size fraction $i$ ($AS_i$) in percent is calculated by

$$AS_i = 100\left(\frac{w_{i,i} - s_i}{W_{ai} - s_i}\right) \tag{2}$$

where $s_i$ is the weight of sand particles larger than the aperture size of sieve $i$.

The total weight of aggregates that have passed through sieve $i$ is $D_i$ (*Fig.1*), where $D_i$ is calculated by

$$D_i = \sum_{k=1}^{i} (W_{ak} - W_{bk}) \tag{3}$$

The weight of ruptured aggregates are related to $D_i$ by

$$D_1 = b_{1,2} + b_{1,3} + b_{1,4} \tag{4}$$

$$D_2 = b_{1,3} + b_{1,4} + b_{1,2,3} + b_{1,2,4} + b_{2,3} + b_{2,4} \tag{5}$$

$$D_3 = b_{1,4} + b_{1,2,4} + b_{1,3,4} + b_{1,2,3,4} + b_{2,4} + b_{2,3,4} + b_{3,4} \tag{6}$$

In the three independent equations Eq. (4) to (6), there are 11 unknowns: $b_{1,2}$, $b_{1,3}$, $b_{1,4}$, $b_{1,2,3}$, $b_{1,2,4}$, $b_{1,3,4}$, $b_{1,2,3,4}$, $b_{2,3}$, $b_{2,4}$, $b_{2,3,4}$ and $b_{3,4}$. To reduce the number of unknowns, it is assumed that ruptured aggregates of the same size fraction (same sieve) have equal weights, provided they are from the same origin (*i.e.*, they are pieces from the same original aggregates). This assumption means

$$b_{i,j} = b_{i,\ldots,j} \tag{7}$$

Thus

$$b_{1,3} = b_{1,2,3}$$
$$b_{1,4} = b_{1,3,4} = b_{1,2,4} = b_{1,2,3,4}$$
$$b_{2,4} = b_{2,3,4}$$

The validity of Eq. (7) is uncertain; however, model testing done later in this study would determine if the use of Eq. (7) is at least reasonable.

Using Eq. (7), the weight of ruptured aggregates are now related to $D_i$ by

$$D_1 = b_{1,2} + b_{1,3} + b_{1,4} \tag{8}$$

$$D_2 = 2\left(b_{1,3} + b_{1,4}\right) + b_{2,3} + b_{2,4} \tag{9}$$

$$D_3 = 4b_{1,2} + 2b_{2,4} + b_{3,4} \tag{10}$$

which can be summarised to $D_i = \sum_{k=1}^{i}\left(2^{i-k}\sum_{j=i+1}^{n} b_{k,j}\right)$. The number of unknowns in the three equations have now been reduced to six: $b_{1,2}$, $b_{1,3}$, $b_{1,4}$, $b_{2,3}$, $b_{2,4}$ and $b_{3,4}$. To further reduce the number of unknowns, it is assumed that aggregates of the same size fraction would break down equally in percentage. This assumption is reasonable because aggregates of the same size fraction belonging to the same soil sample can be assumed to breakdown equally in percentage. Using this assumption means that

$$\frac{b_{1,3}}{b_{1,2}} = \frac{b_{2,3}}{W_{a2}(\text{or } b_{2,2})} \tag{11}$$

$$\frac{b_{1,4}}{b_{1,3}} = \frac{b_{2,4}}{b_{2,3}} = \frac{b_{3,4}}{W_{a3}(\text{or } b_{3,3})} \tag{12}$$

$$\frac{b_{1,4}}{b_{1,2}} = \frac{b_{2,4}}{W_{a2}(\text{or } b_{2,2})} \tag{13}$$

which can be summarised to $\frac{b_{p,q}}{b_{p,r}} = \frac{b_{s,q}}{b_{s,r}}$. By substituting Eq. (11) to (13) into Eq. (9) and (10), it can be shown that Eq. (9) and (10) can be re-expressed to

$$D_2 = \left(2 + \frac{W_{a2}}{b_{1,2}}\right)\left(b_{1,3} + b_{1,4}\right) \tag{14}$$

$$D_3 = 4b_{1,2} + 2W_{a2}\frac{b_{1,4}}{b_{1,2}} + W_{a3}\frac{b_{1,4}}{b_{1,3}} \tag{15}$$

The number of unknowns in the three equations have now been reduced to three: $b_{1,2}$, $b_{1,3}$ and $b_{1,4}$; thus, Eq. (8),(14) and (15) are solvable. The calculated $b_{1,2}$, $b_{1,3}$ and $b_{1,4}$ are substituted back into Eq. (7), and Eq. (11) to (13) to determine $b_{1,2,3}$, $b_{1,2,4}$, $b_{1,3,4}$, $b_{1,2,3,4}$, $b_{2,3}$, $b_{2,4}$, $b_{2,3,4}$ and $b_{3,4}$.

Finally, it can be shown that the general equation to determine the weight of the ruptured aggregates ($b_{1,i} \geq 0$) for any number of sieves is calculated by

$$b_{1,i} = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \qquad (16)$$

where

$$A = -2^{i-1}\delta - \sum_{k=1}^{i-2}\beta_k$$

$$B = \sum_{k=1}^{i-2}\beta_k\left[\lambda + b_{1,(i-k)}\right] - \delta\left\{D_i - 2^{i-1}\left[\lambda - \sum_{k=1}^{i-1}2^{-k}W_{a(k+1)}\right]\right\}$$

$$C = W_{ai}\lambda\delta$$

with

$$\delta = \prod_{j=2}^{i-1} b_{1,j}$$

$$\beta_k = \frac{2^k W_{a(i-k)}\delta}{b_{1,(i-k)}}$$

$$\lambda = D_1 - \sum_{j=2}^{i-1} b_{1,j}$$

As an example: the parameters $A$, $B$ and $C$ for $b_{1,3}$ would be ($-2W_{a2} - 4b_{1,2}$), ($2D_1W_{a2} + 4D_1b_{1,2} - D_3b_{1,2} - 2W_{a2}b_{1,2} - W_{a3}b_{1,2} - 4b^2_{1,2}$), and ($D_1W_{a3}b_{1,2} - W_{a3}b^2_{1,2}$), respectively.

## MATERIALS AND METHODS

Five soil series of various textures and land use were used to test the models (Table 1). Soil samples were taken from several Universiti Putra Malaysia (UPM) farms, and at two soil depths: 0-150 and 150-300 mm. All soil samples were taken randomly in the field, mixed and air-dried for a week. Whole soils (< 2 mm) were analysed for particle size distribution using the pipette method (Gee and Bauder 1986). The soil samples were also dry-sieved into six aggregate size fractions: 5-8, 3-5, 2-3, 1-2, 0.5-1 and 0.3-0.5 mm.

To determine the actual breakdown and distribution of individual aggregate size fractions, 20 g of each aggregate size fraction for each soil sample was wet-

TABLE 1
Physical properties of whole soils used to test the models

| Soil code | Soil series | Taxonomy | Land use | Particle size distribution (%) | | |
|---|---|---|---|---|---|---|
| | | | | clay < 2 μm | silt 2-20 μm | sand > 50 μm |
| SO1 (top) | Munchong | Typic Hapludox | Rubber | 42.58 | 24.92 | 32.50 |
| SO1 (sub) | Munchong | Typic Hapludox | Rubber | 51.57 | 22.44 | 25.99 |
| SO2 (top) | Munchong | Typic Hapludox | Oil palm | 56.48 | 11.32 | 32.20 |
| SO2 (sub) | Munchong | Typic Hapludox | Oil palm | 63.30 | 10.00 | 26.70 |
| SO3 (top) | Melaka | Xanthic Hapludox | Pine | 28.80 | 23.50 | 47.70 |
| SO3 (sub) | Melaka | Xanthic Hapludox | Pine | 39.40 | 25.07 | 35.53 |
| SO4 (top) | Prang | Typic Hapludox | Bare | 62.92 | 18.41 | 18.67 |
| SO4 (sub) | Prang | Typic Hapludox | Bare | 51.75 | 25.94 | 22.31 |
| SO5 (top) | Bungor | Typic Paleudult | Grassland | 22.75 | 7.46 | 69.79 |
| SO5 (sub) | Bungor | Typic Paleudult | Grassland | 23.63 | 8.82 | 67.55 |
| SO6 (top) | Serdang | Typic Paleudult | Vegetable | 34.08 | 11.81 | 54.11 |
| SO6 (sub) | Serdang | Typic Paleudult | Vegetable | 32.13 | 14.18 | 53.69 |

sieved separately using the nested sieves (Yoder 1936; Kemper and Chepil 1965). Before wet-sieving, all aggregates were pre-wetted by incubation under room temperature and at 98% relative humidity for 24 h. Wet-sieving was for 30 minutes, at 40 rpm and through a vertical distance of 40 mm. After wet-sieving, aggregates retained in every sieve were collected separately, oven-dried at 105∞C for 24 h and weighed.

The root mean square error (RMSE) of estimation was calculated by

$\sqrt{\sum_{k=1}^{N}(P_k - O_k)^2 / N}$ where $N$ is the number of observations, and $P_k$ and $O_k$ are the estimated and actual retained aggregates $w_{p,\ldots,j}$ for the $k$th observation, respectively.

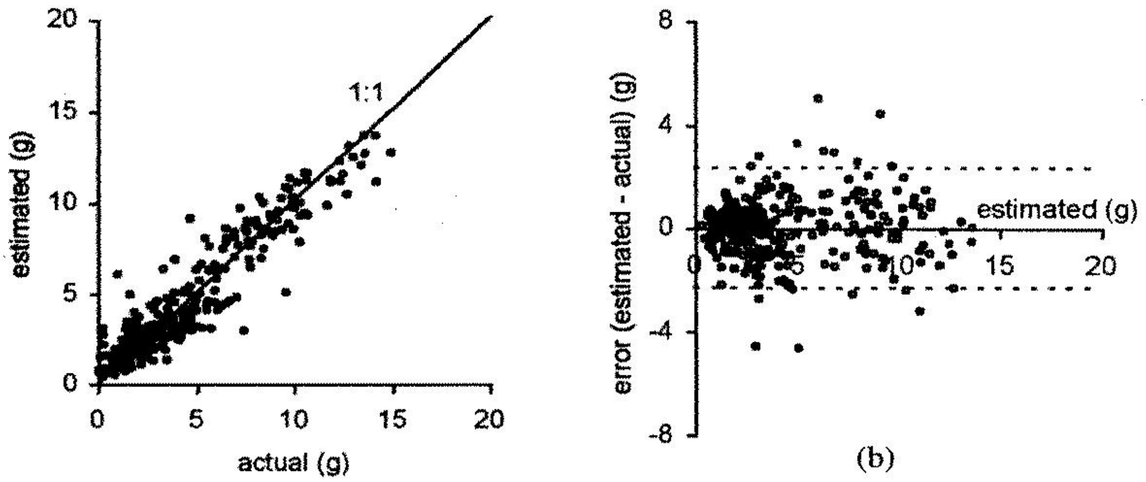


*Fig. 2: Comparisons between the estimated and actual weight of retained aggregates $w_{i,}$ for all soils, and (b) the scatter of the estimation errors. Note: the horizontal dashed lines are the 95% error band (two standard deviations).*

## RESULTS AND DISCUSSION

There was overall good agreement between the model estimates and measured weight of retained aggregates $w_{i,...,j}$ (*Figs. 2a* and *3*). There was a close or tight clustering of points along the 1:1 line of agreement, and the mean estimation error, or RMSE, for the model was small (1.33 g or 6.65%). In addition, the estimation errors were within a narrow 95% error range (two standard deviations) of ± 2.31 g (*Fig. 2b*).

The model also showed overall good agreement between the estimates and actual weight of intact aggregates $w_{i,i}$ (*Fig.4*). As shown by Eq. (2), $w_{i,i}$ must be determined to calculate the stability of aggregate size fraction $i$. The mean error for estimating $w_{i,i}$ was 1.22 g or 6.12%. The model also showed no bias in estimating the weight of intact aggregates $w_{i,i}$, and paired sample $t$-test revealed that the differences between the model estimates and the actual $w_{i,i}$ were not significant at the 5% level ($t = -1.424$; $p < 0.160$).

The accurate model estimates suggested that the use of Eq. (7) was at least reasonable though it would be difficult to determine by experimentation if the ruptured aggregates of the same size fraction and from the same origin (*i.e.*, from the same original aggregates) have approximately equal weights. The model developed in this study saves an enormous amount of work and time because one does not have to wet-sieve each aggregate size fraction separately to determine the individual aggregate breakdown and distribution in the nested sieves. The model showed good accuracy for several different soils despite not requiring any information about the properties of the soil (*e.g.* water content, aggregate size and texture) and wet-sieving method (*e.g.*, pre-wetting treatment, duration of wet-sieving and amount of water used). This is in contrast with the requirements by other empirical equations (*e.g.* Zanini *et al.* 1998). What matters ultimately for the model are the weights of aggregates before and after wet-sieving in each sieve, and the validity of Eq. (7) and the assumption of a simultaneous aggregate breakdown (*Fig. 1*).

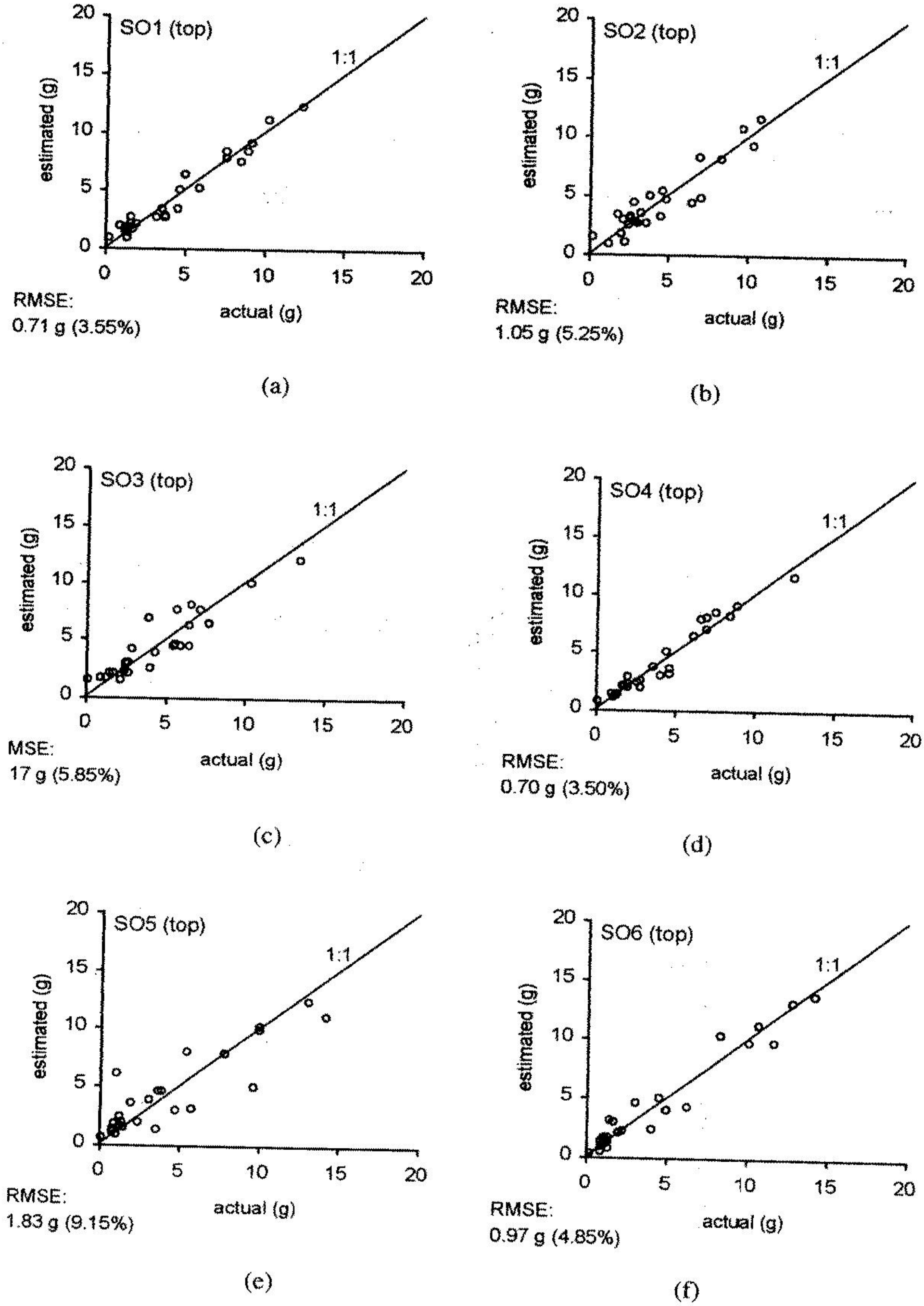


*Fig. 3: Comparisons between the estimated and actual weight of retained aggregates $w_{i,...,j}$ for each soil type (0-150 mm soil depth).*

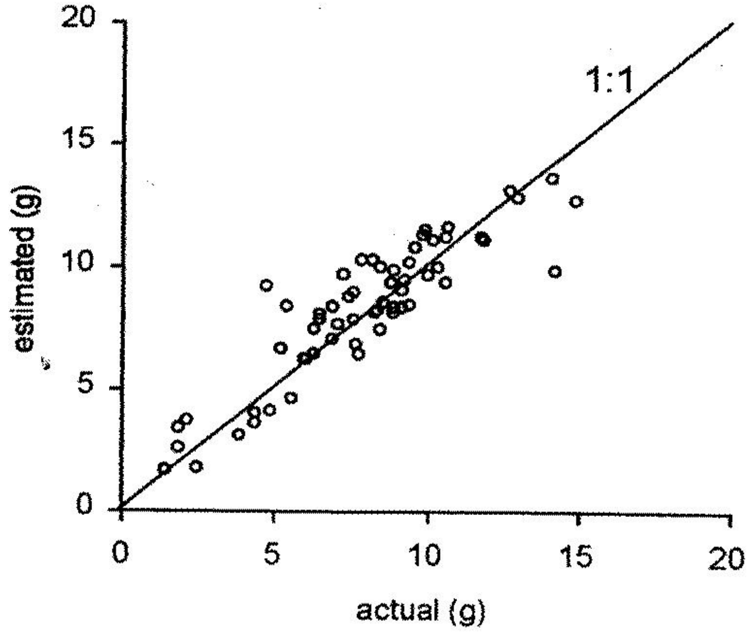


*Fig. 4: Comparisons between the estimated and actual weight of intact aggregates $w_{i,i}$ for all soils*

## CONCLUSION

This study successfully developed an accurate model to estimate the amount of breakdown and distribution of aggregates in the usual wet-sieving method (using nested sieves). Tested on several soil types, the model showed good accuracy, having a mean estimation error of 1.33 g or 6.65%. The model also showed no bias in estimation.

The advantage of this study's model was that it was fully mechanistic, requiring no empirical coefficients or calibration, and gave accurate results despite not requiring any information about the properties of the soils or wet-sieving method. The model saves an enormous amount of work and time because one does not have to wet-sieve each aggregate size fraction separately to determine the individual aggregate breakdown and distribution in the nested sieves.

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